Constructive action of noise for impulsive noise removal in scalar images

A. Histace and D. Rousseau

We propose a nonlinear variational approach to remove impulsive noise in scalar images. Taking benefit from recent studies on the use of stochastic resonance and the constructive role of noise in nonlinear processes, our process is based on the classical restoration process of Perona-Malik in which a Gaussian noise is purposely injected. We show that this new process can outperform the original restoration process of Perona-Malik.

Aim and motivation : Removing impulsive noises from scalar images is a problem of great interest since these short duration and high energy noises can degrade the quality of digital images in a large variety of practical situations [1]. In this context of non Gaussian noise, nonlinear processes are often invoqued. Among these nonlinear processes median filtering is a classical tool leading to good results [2]. Nevertheless, these median filtering techniques involves strong statistics calculation and can turn out to be highly time consuming to compute. Another nonlinear process classically used for restoration tasks is the diffusion process of Perona-Malik [3]. This process, based on a variational approach, presents short implementation time and has the ability to remove noises while keeping edges stable on many scales. The Perona-Malik process also have its own limitations. Among these limitations, the smoothing property of the diffusion process does not preserve the information present in area with texture or small but significative gradients [4]. As paradoxical as it may seem, to limit the effect of this drawback, we propose a new variant of the Perona-Malik process in which a controlled amount of noise is injected in the nonlinear process. The possibility of constructive action of noise in nonlinear processes is now a well established paradigm known under the name of stochastic resonance (see [5] for a recent overview in electronic). Up to now, this paradigm has essentially been illustrated with monodimensional signals. This work is a new feature of noise enhanced information processing presented here for the first time in the context of image restoration.

Method : Let ψ_{ori} denotes an original image and ψ_0 denotes the same image corrupted by an input impulsive noise ξ imposed by the external environment :

$$\psi_0(x, y) = \psi_{ori}(x, y) + \xi(x, y) \; . \tag{1}$$

The restoration of ψ_0 aims at the removal of ξ from ψ_0 to obtain an image as similar as possible to ψ_{ori} of Eq. (1).

The Perona-Malik's restoration approach of ψ_0 is equivalent to an iterative minimization problem [6], solved by the resolution of the Partial Differential Equation (PDE) given by :

$$\frac{\partial \psi(x, y, t)}{\partial t} = div(g(|\nabla \psi(x, y, t)|)\nabla \psi(x, y, t)), \qquad \psi(x, y, t = 0) = \psi_0 \; , \tag{2}$$

where $g(.)$ is a nonlinear decreasing function of the gradient (∇) of ψ the restored image at a scale t (which can be interpreted as a time evolution parameter) and div the divergence operator. For practical numerical implementation, the process of Eq. (2) is discretized with a time step τ. The images $\psi(t_n)$ are calculated, with Eq. (2), at discrete instant $t_n = n\tau$ with n the number of iterations in the process. We are going to compare the standard Perona-Malik process of Eq. (2) with the following diffusion process

$$\frac{\partial \psi(x, y, t)}{\partial t} = div(g_\eta(|\nabla \psi(x, y, t)|)\nabla \psi(x, y, t)) \; , \tag{3}$$

in which the nonlinear function $g(.)$ in Eq. (2) has been replaced by $g_\eta(.)$ with

$$g_\eta(u(x, y)) = \frac{1}{M} \sum_{i=1}^{M} g(u(x, y) + \eta_i(x, y)) \; , \tag{4}$$

where η_i functions are M independent noises assumed independent and identically distributed with probability density function (pdf) f_η and rms amplitude σ_η. The noises η_i which are purposely added noises applied to influence the operation of the $g(.)$ has to be clearly distinguished from the input noise ξ of Eq. (1) which is considered as a noise imposed by the external environment that we wish to remove. The choice of g_η in Eq. (4) is inspired from recent studies on the constructive action of noise in parallel arrays of nonlinear electronic devices [7] and transposed here in the domain of image processing. The quality of the restored image $\psi(t_n)$ at a given instant t_n is assessed by the normalized crosscovariance $C_{\psi_{ori}\psi(t_n)}$ given by :

$$C_{\psi_{ori}\psi(t_n)} = \frac{\langle(\psi_{ori} - \langle\psi_{ori}\rangle)(\psi(t_n) - \langle\psi(t_n)\rangle)\rangle}{\sqrt{\langle(\psi_{ori} - \langle\psi_{ori}\rangle)^2\rangle\langle(\psi(t_n) - \langle\psi(t_n)\rangle)^2\rangle}} \quad , \tag{5}$$

where $\langle..\rangle$ denotes the spatial average.

Results : For illustration of the processes of Eqs. (2) and (3), the image "cameraman" (see image A in Fig. 2), which presents strong and small gradients, textured and non textured regions of interest, has been taken as reference image in this study.

The original nonlinear function $g(.)$ proposed by Perona-Malik in [3], with $g(u(x,y)) = e^{-\frac{|u(x,y)|}{k^2}}$, is chosen in this study. The pdf f_η of the M noises η_i in Eq. (3) are chosen Gaussian. Other measures of similarity (like the Peak Signal to Noise Ratio) ,images and pdf for η_i have been tested ; Results were quantitatively and qualitatively similar to the ones presented below. In Fig. 1, the similarity between the restored and original image, assessed by the normalized crosscovariance of Eq. (5), overpass the classical Perona-Malik process for all values tested for M. This demonstrates the possibility of improving the performance of the Perona-Malik process by injecting a non zero amount of the M noises η_i in Eq. (4). Moreover, in Fig. 1, one can also notice that the convergence speed of the diffusion process is increased with the presence of the M noises η_i. This acceleration of the convergence is another benefit

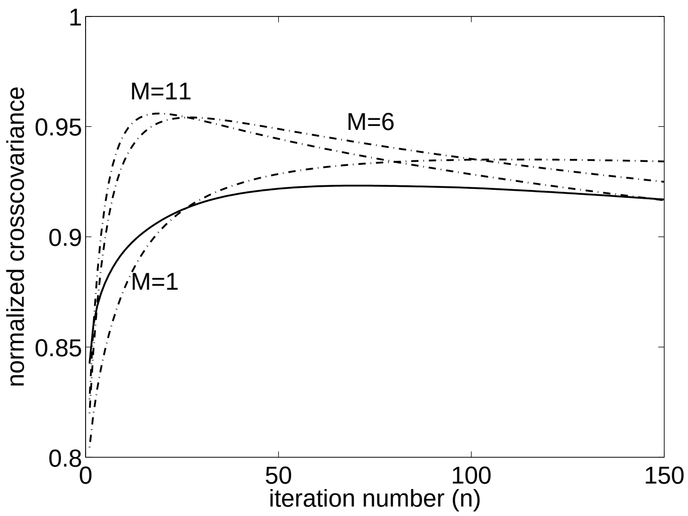

Fig. 1. Normalized crosscovariance of Eq. (5) as a function of the iteration number n of the restoration process. Dash-dotted lines stand for our modified version of the Perona-Malik process of Eqs. (3) and (4) for various number M of independent noises η_i. Solid line stands for the classical Perona-Malik process of Eq. (2). The rms amplitude σ_η of the M noises η_i is fixed to $\sigma_\eta = 0.7$. The parameter k in the nonlinear function $g(.)$ and the step time τ, characterizing the convergence speed of the diffusion process, are fixed to $k = 0.2$ and $\tau = 0.25$ in both Eqs. (2) and (3).

obtained from the purposely injected noises in Eqs. (3) and (4) by comparison with the classical Perona-Malik's process of Eq. (2). As visible in Fig. 2, the similarity improvement

Fig. 2. Visual comparison of the performance of the restoration processes by Eq. (2) and Eq. (3). A : the original 'cameraman' image ψ_{ori} of Eq. (1). B : the noisy version ψ_0 of ψ_{ori} corrupted by an additive salt and pepper noise ξ (Eq. (1)) of standard deviation 0.1 ; C and D are respectively obtained with Eq. (2) and Eq. (3) for the optimal number of iteration n corresponding to the highest value of the normalized crosscovariance in Fig. 1 ($M = 11$ in the case of D).

shown in Fig. 1 is also perceptible with the images themselves. The addition of noise leads to a better contrast and a better preservation of the structure which are characterized by small gradient (for example the buildings on the background is more visible and the texture of the grass is preserved by comparison with the classical Perona-Malik process). In Fig. 3, the number of iteration n of the diffusion processes of Eqs. (3) and (4) is fixed. The normalized crosscovariance of Eq. (5) culminates at a maximum for an optimal nonzero noise level of the M noises injected in Eq. (4). This clearly demonstrates the possibility of a constructive role of noise in the diffusion process of Eqs. (3) and (4) for an image restoration task.

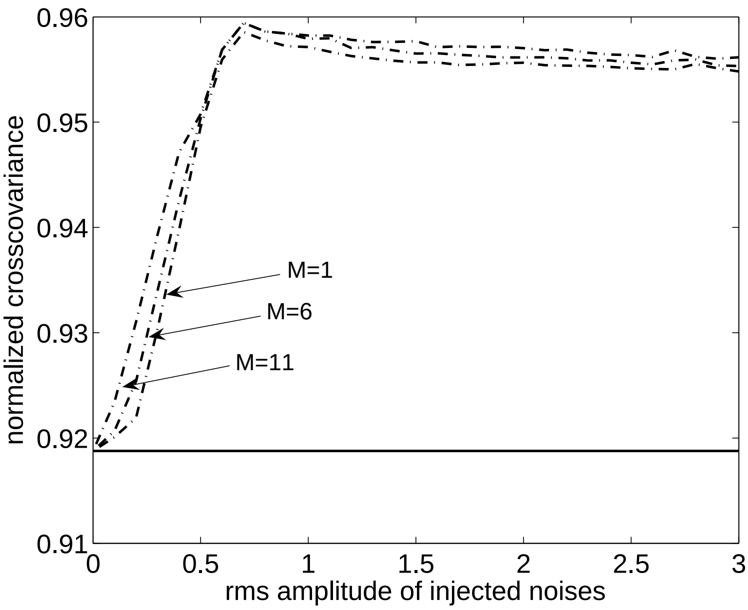

Fig. 3. Normalized crosscovariance of Eq. (5) as a function of the rms amplitude σ_η of the noise purposely injected in Eq. (4). Dash-dotted lines stand for our modified version of the Perona-Malik process of Eqs. (3) and (4) for various number M of independent noises η_i. Solid line stands for the classical Perona-Malik process of Eq. (2). The number of iteration n is fixed for all the lines to $n = 75$.

A. Histace and D. Rousseau *Laboratoire d'Ingénierie des Systèmes Automatisées (LISA CNRS FRE 2656), 62 avenue Notre Dame du Lac, 49000 ANGERS France*, *aymeric.histace@univ-angers.fr*.

References

1. C. G. Boncelet, *Image Noise Models. In : Bovik A, editor. Handbook of Imageand Video Processing.* New York : Academic Press, 2000.
2. O. Maheshwari and D. Ebenezer, "Simultaneous removal of positive and negative impulses in images by using adaptive length recursive weighted median filter," *WSEAS Transactions and Communications*, vol. 4, no. 12, pp. 1350–1355, 2005.
3. P. Perona and J. Malik, "Scale-space and edge detection using anistropic diffusion," *IEEE Transcations on Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629–639, 1990.
4. F. Catte, T. Coll, P. Lions, and J. Morel, "Image selective smoothing and edge detection by nonlinear diffusion," *SIAM Journal of Applied Mathematics*, vol. 29, no. 1, pp. 182–193, 1992.
5. G. P. Harmer, B. R. Davis, and D. Abbott, "A review of stochastic resonance : Circuits and measurement," *IEEE Transactions on Instrumentation and Measurement*, vol. 51, pp. 299–309, 2002.
6. P. Komprobst, R. Deriche, and G. Aubert, "Image sequence analysis via partial differential equations," *Journal of Mathematical Imaging and Vision*, vol. 11, no. 1, pp. 5–26, 1999.
7. F. Chapeau-Blondeau and D. Rousseau, "Enhancement by noise in parallel arrays of sensors with power-law characteristics," *Physical Review E*, vol. 70, pp. 060 101(R),1–4, 2004.