NOISE–ENHANCED ANISOTROPIC DIFFUSION FOR SCALAR IMAGE RESTORATION

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ABSTRACT

We demonstrate the possibility of improving the standard Perona-Malik's anisotropic diffusion process for image restoration thanks to a constructive action of a purposely injected noise. The effect is shown to be robustly preserved for various types of native noise when applied to textured images. This is interpreted as a novel instance of the phenomenon of stochastic resonance or improvement by noise in image processing.

1. A STOCHASTIC VARIANT OF PERONA–MALIK'S PROCESS

We consider an original image ψ_{ori} coupled with a noisy component ξ which degrades the observable image ψ_0. Our goal is to remove the noise component ξ of ψ_0 in order to obtain an image as similar as possible to ψ_{ori}. We propose to tackle this standard restoration task with a stochastic variant of Perona–Malik's process, recently introduced in [1], that we briefly describe here. The original Perona–Malik's process [2] is an anisotropic diffusion process inspired from the physics of temperature diffusion in which the observable noisy image ψ_0 is restored by considering the solution of the partial differential equation given by

$$\frac{\partial \psi}{\partial t} = div(g(\|\nabla\psi\|)\nabla\psi), \qquad \psi(x,y,t=0) = \psi_0 \ , \quad (1)$$

where the anisotropy of this diffusion process is governed by $g(.)$ a nonlinear decreasing function of the norm of the gradient $\nabla\psi$. In this study, we consider the process given by

$$\frac{\partial \psi}{\partial t} = div(g_\eta(\|\nabla\psi\|)\nabla\psi) \ , \quad (2)$$

which is of a form similar to Eq. (1) except for the nonlinear function $g_\eta(.)$ which is given by

$$g_\eta(u) = g(u + \eta(x,y)) \ , \quad (3)$$

where η is a noise assumed independent and identically distributed with probability density function (pdf) $f_\eta(u)$ and rms amplitude σ_η. The noise η, which is distinct from the native noise component ξ to be removed, is a purposely added noise applied to influence the operation of $g(.)$. In [1], we have shown that this injection of noise in Eq. (3) can improve the restoration process by comparison with standard Perona–Malik's process of Eq. (1) when the native noise component ξ is an impulsive noise.

In this study, we consider the stochastic variant of Perona–Malik's process given by Eqs. (2) and (3), and we investigate the possible extension of the previous results obtain in [1] to other types of native noises ξ, and the robustness of this constructive action of the noise.

2. NOISE–ENHANCED PERFORMANCE

For illustration, we choose the original nonlinear function $g(.)$ proposed by Perona–Malik in [2], given by

$$g(u) = e^{-\frac{\|u\|^2}{k^2}} \ , \quad (4)$$

where parameter k can be seen as a soft threshold controlling the decrease of $g(.)$ and the amplitude of the gradients to be preserved from the diffusion process. The pdf $f_\eta(u)$ of the noise η in Eq. (2) is chosen Gaussian. We choose to assess the performance of the restoration processes with the normalized cross-covariance $C_{\psi_{ori}\psi(t_n)}$,

$$C_{\psi_{ori}\psi(t_n)} = \frac{\langle(\psi_{ori} - \langle\psi_{ori}\rangle)(\psi(t_n) - \langle\psi(t_n)\rangle)\rangle}{\sqrt{\langle(\psi_{ori} - \langle\psi_{ori}\rangle)^2\rangle\langle(\psi(t_n) - \langle\psi(t_n)\rangle)^2\rangle}} \ , \quad (5)$$

with $\langle..\rangle$ a spatial average, $\psi(t_n)$ the images calculated with Eqs. (1) or (2) at discrete instants $t_n = n\tau$ where n is the number of iterations in the process and τ the time step used to discretize Eqs. (1) and (2). The image "cameraman" (see image (d) in Fig. 1), is chosen as reference for the original image ψ_{ori}. Noisy versions of this original image are presented as the observable images ψ_0 of our restoration task in Fig. 1. The 3 observable images (a,b,c) of Fig. 1, which present the same level of similarity (assessed by the normalized cross-covariance of Eq. (5)) with the original image ψ_{ori}, have respectively been corrupted by an additive, a multiplicative and an impulsive noise component ξ.

We are now in position of comparing the restoration of the noisy images (a,b,c) of Fig. 1 by the classical Perona–Malik's process of Eq. (1) and our stochastic version of this process. As noticeable in Fig. 2, the similarity between the restored and original image, assessed by the normalized cross-covariance of Eq. (5), overpasses the classical Perona–Malik's process only when the native noise ξ is impulsive.

(a)

(b)

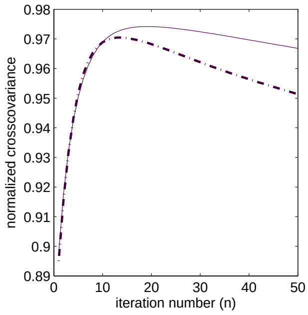
(c)

(d)

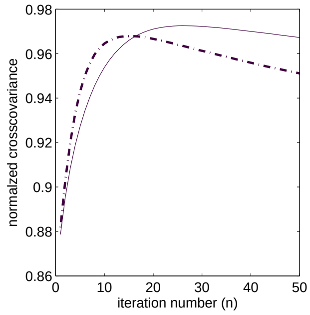
Fig. 1. The original image ψ_{ori} cameraman (d) corrupted by 3 different native noises ξ: (a) additive zero-mean Gaussian noise with $\psi_0 = \psi_{ori} + \xi$, (b) multiplicative Gaussian noise of mean unity with $\psi_0 = \psi_{ori} + \xi.\psi_{ori}$, (c) impulsive noise. The rms amplitude of these noises are separately adjusted in order to have each of the images (a,b,c) characterized by the same normalized cross-covariance (given in Eq. (5)) with the original image equal to 0.87.

A subjective visual comparison of the performance of the processes of Eqs. (1) and (2) is also given in Fig. 3. By contrast with the results obtained with the normalized cross-covariance, in Fig. 3, images restored by the process of Eq. (2) appear to be of better visual interest than those obtained with the classical Perona–Malik's process of Eq. (1) for all the 3 types of noise component tested. This is especially visible, in Fig. 3, in areas of the "cameraman" image characterized by small gradients (face, buildings in the background, or textured area like grass) which are preserved from the diffusion process and better restored with the presented stochastic approach of Eq. (2) than with the classical Perona–Malik's process of Eq. (1).

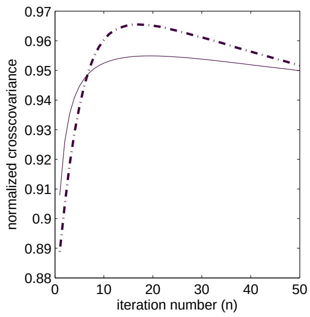
In order to provide a quantitative validation of this visual observation with an objective measure of similarity, we propose to study the evolution of the normalized cross-covariance for both processes of Eq. (1) and Eq. (2) when a textured region of interest (see image (d) in Fig. 4) is extracted from the "cameraman"image. As visible in Fig. 4, the stochastic version of the Perona–Malik's process outperforms the classical version of this process for all the noise components tested.



(a)



(b)



(c)

Fig. 2. Quantitative performance of the restoration processes by Eq. (1) and Eq. (2) for the 3 observable images (a,b,c) of Fig. 1 respectively standing for the (a,b,c) graphes of this figure. Each graph shows the normalized cross-covariance of Eq. (5) as a function of the iteration number n of the restoration process. Dash-dotted lines stand for our stochastic version of the Perona–Malik's process of Eqs. (2) and (3). Solid lines stand for the classical Perona–Malik's process of Eq. (1). Parameter k in the nonlinear function $g(.)$ and time step τ, characterizing the convergence speed of the diffusion process, are fixed to $k = 0.1$ and $\tau = 0.2$ in both Eqs. (1) and (2). The standard deviation σ_η of the purposely injected noise is fixed to 0.2.

A complementary analysis is presented in Fig. 5 where the number of iteration n of the diffusion process of Eq. (2) is fixed. The evolution of the normalized cross-covariance of Eq. (5) is then presented as a function of the rms amplitude σ_η of the Gaussian noise purposely injected in Eq. (3). As visible in Fig. 5, the normalized cross-covariance of Eq. (5) experiences, for all the 3 tested noise components, a nonmonotonic evolution and culminates at a maximum for an optimal nonzero level of the Gaussian noise injected in Eq. (3). All these results demonstrates that the possibility of improving the performance of the Perona–Malik's process by injecting a non zero amount of the noise η in Eq. (3) is not restricted to impulsive noise but can be extended to other noise coupling like multiplicative or additive noise components.

At last, we investigate the robustness of the noise enhanced image restoration demonstrated in this section with respect to

Fig. 3. Visual comparison of the performance of the restoration processes by Eq. (1) and Eq. (2) with the same conditions of Fig. 2. The left column shows the results obtained with usual Perona–Malik's restoration process of Eq. (1) and the right column with our stochastic version of the Perona–Malik's process of Eq. (2). Each image is obtained with the iteration number n corresponding to the highest value of the normalized cross-covariance of Fig. 2. The top, middle and bottom lines are respectively standing for the additive, multiplicative and impulsive noise component described in Fig. 1.

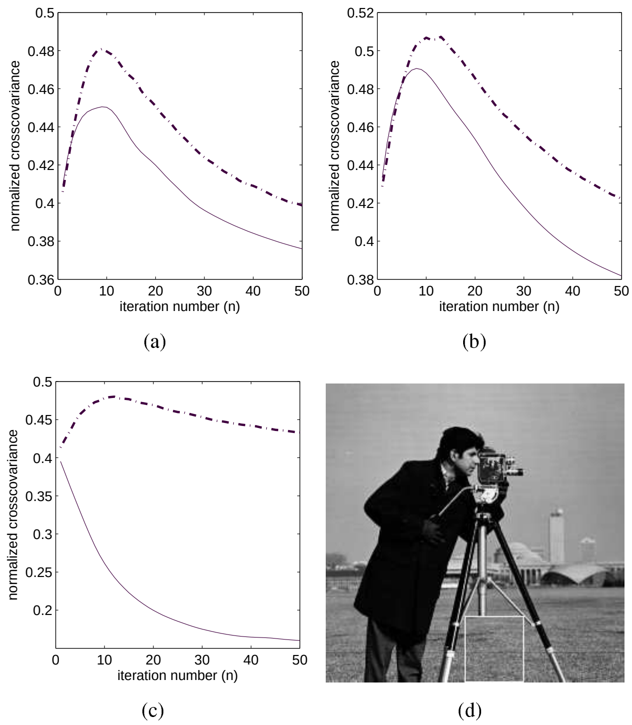


Fig. 4. Same as in Fig. 2 except that the analysis is restricted to a textured area of the "cameraman" image delimited by a solid line in (d).

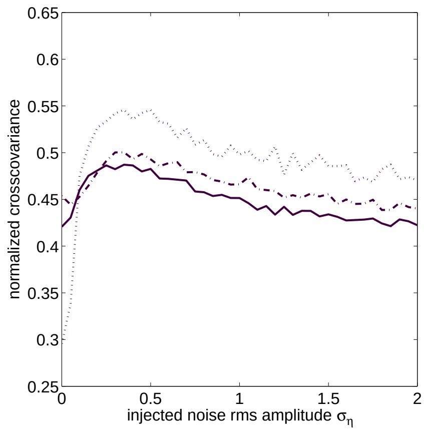


Fig. 5. Same as in Fig. 4 except that the normalized cross-covariance of Eq. (5) is here plotted as a function of the rms amplitude σ_η of the Gaussian noise η purposely injected in Eq. (3) with the number of iteration n which is fixed to $n = 15$. Solid, dash-dotted and dotted lines are respectively standing for the additive, multiplicative and impulsive noise components described in Fig. 1

the choice of parameter k in $g(.)$ of Eq. (4). As visible in Fig. 6, for large amount of injected noise, the constructive action of the noise in Eqs. (1) or (2) does not critically depend on the choice of parameter k. This is by contrast with standard Perona–Malik's process which does not present this robustness property. The image chosen as reference in Fig. 6 is the "D57" textured image extracted from the Brodatz's databank. Multiple other configurations have been tested with variations concerning the nonlinear function $g(.)$ of Eq. (4), the injected noise η of Eq. (3), the measure of similarity (like SNR) and the reference image, and they all demonstrate the same possibility of a robust constructive action of the noise to improve classical Perona–Malik's process.

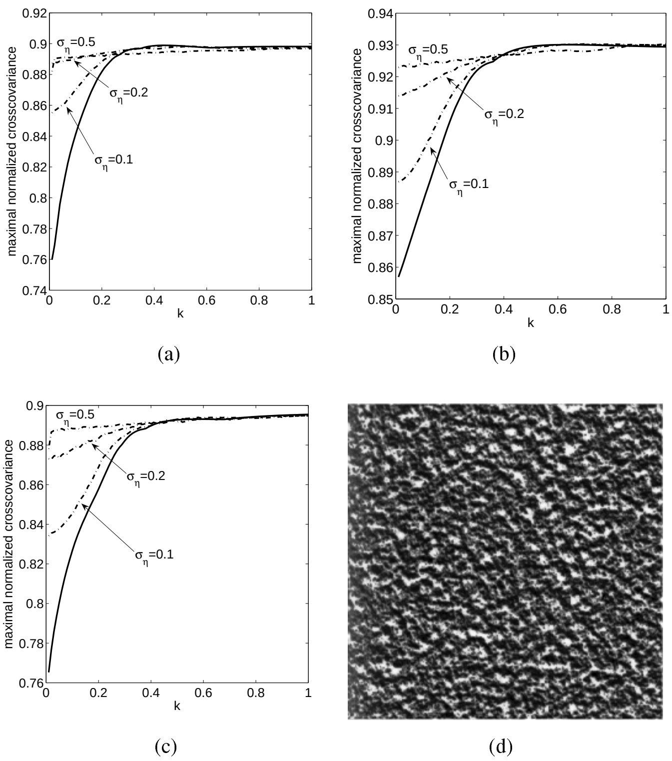


Fig. 6. Normalized cross-covariance of Eq. (5) plotted as a function of the parameter k of the nonlinear function $g(.)$ for various rms amplitude σ_η of the Gaussian noise η purposely injected in Eq. (3). The number of iteration n is fixed at the maximum of the normalized cross-covariance. Same designation as in Fig. 2 for the solid and dashed dotted lines. The original Brodatz texture"D57" (d) is respectively corrupted by (a) an additive zero-mean Gaussian noise with $\psi_0 = \psi_{ori} + \xi$, (b) a multiplicative Gaussian noise of mean unity with $\psi_0 = \psi_{ori} + \xi.\psi_{ori}$, (c) an impulsive noise.

3. DISCUSSION

By showing the robustness of the stochastic version of Perona–Malik's process, this study contributes to extend the interest of the results presented in [1], which were restricted to the case of an impulsive noise with a fixed parameter k, to other fields of imaging. Therefore, this process may find applicability in the restoration of textures obtained from imaging systems corrupted by thermal noise (widely modeled by an additive Gaussian noise) or from coherent imaging systems (SONAR, SAR, or LASER) where the images are corrupted by a multiplicative noise (i.e. speckle noises) [3].

Beyond practical perspectives, our process can be seen as a novel instance of stochastic resonance. Introduced some twenty years ago in the context of nonlinear physics [4], stochastic resonance has gradually been reported, under various forms, in a still-increasing variety of processes (see for example [5] for a review in physics, [6] for an overview in electrical engineering and [7] for the domain of signal processing). Stochastic resonance can be described as the possibility of improving the situation of some information-bearing quantity, thanks to the action of an independent noise. This is clearly the paradigm established here, as a proof of feasability, for an image restoration task. Up to now, stochastic resonance or noise aided signal processing has essentially been reported for mono-dimensional signal processing tasks like detection or estimation [7]. A classical situation relevant to stochastic resonance is obtained when a small information-carrying signal is by itself too weak to elicit an efficient response from a nonlinear system (presenting for example a threshold) [8]. The noise then cooperates constructively with the small signal, in such a way as to elicit a more efficient response from the nonlinear system (noise helps the signal to reach the threshold in the example). This cooperative effect usually exhibits a maximum at an optimum noise level beyond which the noise becomes too strong (noise can reach the threshold by itself in the example). The constructive action of noise reported here for an image processing task presents some similarities with this usual mechanism of stochastic resonance. In our case, the nonlinearity comes from the function $g(.)$ of Eq. (4) for which parameter k somehow plays the role of a soft threshold. A small information-carrying gradient, too weak to reach the soft threshold presented by $g(.)$, is diffused by the standard Perona–Malik's process. Therefrom, like in the classical stochastic resonance mechanism, the noise is found to cooperate constructively with the small gradient to reach the soft threshold in order to be preserved from the diffusion process. As illustrated in Fig. 5 when raising the level of the noise injected in the nonlinear process, a point beyond which the noise becomes too strong is reached and the cooperative effects demonstrated here exhibits the classical signature of stochastic resonance. Various other types of stochastic resonance have been studied with monodimensional signals and it may be interesting to examine in detail how they could be transposed in a nontrivial way (i.e. not in pixel to pixel process) to image processing.

Concerning anisotropic diffusion applied to image processing, it has been widely developed (see [9] for an overview) for image restoration since its introduction by Perona and Malik in [2], with more complex anisotropic diffusion processes. They have in common with Perona–Malik's process, chosen here for its simplicity, to involve a nonlinear process inspired from the physics of temperature diffusion. [10] and [11], for example, can be considered as extensions of Perona–Malik's process [2]. Both methods still involve a nonlinear process driven by the nonlinear function $g(.)$ of Eq. (4). [10] computes the calculation of $g(.)$ on the gradient $\nabla\psi$ of Eq. (1) convolved with a Gaussian kernel with a fixed standard deviation, and [11] computes the same calculation but with a Gaussian kernel with a standard deviation which is function of time step τ used for discretization of the partial differential equation. Introduction of these Gaussian kernels tends to smooth variations of the gradient, and as a consequence, ensures stability of the solution by contrast with Perona–Malik's approach. As a perspective, it will be interesting to study how the constructive action of the noise reported here would operate with these extensions of Perona–Malik's process.

4. REFERENCES

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